Designing parametric spur gears with Catia V5

Since the geometry of a spur gear is controlled by a few parameters, we can design a generic gear controlled by the following parameters:

- The pressure angle \( \theta \).
- The modulus \( m \).
- The number of teeth \( Z \).

This tutorial shows how to make a basic gear that you can freely re-use in your assemblies.

1. Gears theory and standards

1.1 Sources, credits and links

- Most of my tutorial is based on a nice tutorial on helical gears in English at [http://ggajic.sbb.co.yu/pub/catia/](http://ggajic.sbb.co.yu/pub/catia/).
- I improved it a little for making an exactly symmetric tooth.
- The mathematic description of the involute curve is visually explained in French at [http://serge.mehl.free.fr/courbes/developC.html](http://serge.mehl.free.fr/courbes/developC.html).
- The formulas of the involute curve can also be found in French at [http://www.mathcurve.com/courbes2d/developpantedecercle/developpantedecercle.shtml](http://www.mathcurve.com/courbes2d/developpantedecercle/developpantedecercle.shtml).
- The gear technology is explained in French at [http://casm.insa-lyon.fr/engrenag/](http://casm.insa-lyon.fr/engrenag/).
- The conventional formulas and their names in French come from the pocket catalog Engrenages H.P.C, June 1999 edition.

1.2 Table of useful parameters and formulas

Here is a table containing the parameters and formulas used later in this tutorial:

- The table is given first so that you can use it for further copy/paste operations.
- All the units are defined in the metric system.
- This figure shows the \( a \), \( r_a \), \( r_b \), \( r_f \), \( r_p \) parameters defined in the table:
1.3 Notes about the formulas (in French)

Formule N°11: explication de l’équation

\[ rb = rp \times \cos(a) \]

- La crémaillère de taillage est tangente au cercle primitif.
- Au point de contact, \( a \) définit l’angle de pression de la ligne d’action.
- La ligne d’action est tangente au cercle de base.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Type or unit</th>
<th>Formula</th>
<th>Description</th>
<th>Name in French</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>angular degree</td>
<td>20deg</td>
<td>Pressure angle: technologic constant ((10\text{deg} \leq a \leq 20\text{deg}))</td>
<td>Angle de pression.</td>
</tr>
<tr>
<td>2</td>
<td>m</td>
<td>millimeter</td>
<td>—</td>
<td>Modulus.</td>
<td>Module.</td>
</tr>
<tr>
<td>3</td>
<td>Z</td>
<td>integer</td>
<td>—</td>
<td>Number of teeth ((5 \leq Z \leq 200)).</td>
<td>Nombre de dents.</td>
</tr>
<tr>
<td>4</td>
<td>p</td>
<td>millimeter</td>
<td>( m \times \pi )</td>
<td>Pitch of the teeth on a straight generative rack.</td>
<td>Pas de la denture sur une crémaillère génératrice rectiligne.</td>
</tr>
<tr>
<td>5</td>
<td>e</td>
<td>millimeter</td>
<td>( p / 2 )</td>
<td>Circular tooth thickness, measured on the pitch circle.</td>
<td>Epaisseur d’une dent mesurée sur le cercle primitif.</td>
</tr>
<tr>
<td>6</td>
<td>ha</td>
<td>millimeter</td>
<td>—</td>
<td>Addendum = height of a tooth above the pitch circle.</td>
<td>Saillie d’une dent.</td>
</tr>
<tr>
<td>7</td>
<td>hf</td>
<td>millimeter</td>
<td>if ( m &gt; 1.25 ) ( hf = m \times 1.25 ) else ( hf = m \times 1.4 )</td>
<td>Dedendum = depth of a tooth below the pitch circle. Proportionnally greater for a small modulus ((\leq 1.25 \text{mm})).</td>
<td>Creux d’une dent. Plus grand en proportion pour un petit module ((\leq 1.25 \text{mm})).</td>
</tr>
<tr>
<td>8</td>
<td>rp</td>
<td>millimeter</td>
<td>( m \times Z / 2 )</td>
<td>Radius of the pitch circle.</td>
<td>Rayon du cercle primitif.</td>
</tr>
<tr>
<td>9</td>
<td>ra</td>
<td>millimeter</td>
<td>( rp + ha )</td>
<td>Radius of the outer circle.</td>
<td>Rayon du cercle de tête.</td>
</tr>
<tr>
<td>10</td>
<td>rf</td>
<td>millimeter</td>
<td>( rp - hf )</td>
<td>Radius of the root circle.</td>
<td>Rayon du cercle de fond.</td>
</tr>
<tr>
<td>11</td>
<td>rb</td>
<td>millimeter</td>
<td>( rp \times \cos(a) )</td>
<td>Radius of the base circle.</td>
<td>Rayon du cercle de base.</td>
</tr>
<tr>
<td>12</td>
<td>rc</td>
<td>millimeter</td>
<td>( m \times 0.38 )</td>
<td>Radius of the root concave corner. ((m \times 0.38)) is a normative formula.</td>
<td>Congé de raccordement à la racine d’une dent. ((m \times 0.38)) vient de la norme.</td>
</tr>
<tr>
<td>13</td>
<td>t</td>
<td>floating point number</td>
<td>( 0 \leq t \leq 1 )</td>
<td>Sweep parameter of the involute curve.</td>
<td>Paramètre de balayage de la courbe en développante.</td>
</tr>
<tr>
<td>14</td>
<td>yd</td>
<td>millimeter</td>
<td>( rb \times (\sin(t \times \pi) - \cos(t \times \pi) \times t \times \pi) )</td>
<td>Y coordinate of the involute tooth profile, generated by the t parameter.</td>
<td>Coordonnée Y du profil de dent en développante de cercle, générée par le paramètre t.</td>
</tr>
<tr>
<td>15</td>
<td>zd</td>
<td>millimeter</td>
<td>( rb \times (\cos(t \times \pi) + \sin(t \times \pi) \times t \times \pi) )</td>
<td>Z coordinate of the involute tooth profile.</td>
<td>Coordonnée Z du profil de dent en développante de cercle.</td>
</tr>
<tr>
<td>16</td>
<td>ro</td>
<td>millimeter</td>
<td>( rb \times a \times \pi / 180\text{deg} )</td>
<td>Radius of the osculating circle of the involute curve, on the pitch circle.</td>
<td>Rayon du cercle osculateur à la courbe en développante, sur le cercle primitif.</td>
</tr>
<tr>
<td>17</td>
<td>c</td>
<td>angular degree</td>
<td>( \text{sqrt}(1 / \cos(a) - 1) / \Pi \times 180\text{deg} )</td>
<td>Angle of the point of the involute that intersects the pitch circle.</td>
<td>Angle du point de la développante à l'intersection avec le cercle primitif</td>
</tr>
<tr>
<td>18</td>
<td>phi</td>
<td>angular degree</td>
<td>( \text{atan}(yd(c) / zd(c)) + 90\text{deg} / Z )</td>
<td>Rotation angle used for making a gear symetric to the ZX plane</td>
<td>Angle de rotation pour obtenir un roue symétrique par rapport au plan ZX</td>
</tr>
</tbody>
</table>
On a donc un triangle rectangle à résoudre.

**Formule N°12:**
- Entre le cercle de pied et les flancs des dents, prévoir un petit congé de raccordement pour atténuer l'usure en fatigue.

**Formules N°14 et N°15:** explication de $zd = rb \cdot \cos(t) + rb \cdot t \cdot \sin(t)$
- La développante est tracée sur le plan YZ, qui correspond à la vue de face dans Catia.
- Le premier terme $rb \cdot \cos(t)$ correspond à une rotation suivant le cercle de base.
- Le second terme $rb \cdot t \cdot \sin(t)$ correspond au déroulement de la développante.
- Cette expression rappelle que le rayon de courbure de la développante vaut $rb$.

**Formule N°16:**
- Pour simplifier le dessin d'un engrenage, on peut éventuellement remplacer la développante de cercle par un arc de cercle.

A good approximation of a curve at a given point is the osculating circle.

The osculating circle of a curve at a point shares with the curve at that point:
- A common tangent line (continuity of the 1st derivative).
- A common radius of curvature (continuity of the 2nd derivative).

Cercle osculateur à la courbe développante au niveau du diamètre primitif:
- L'angle de la développante est égal à l'angle de pression $a$.
- Le rayon du cercle osculateur est donc: $ro = rb \cdot a \cdot \pi / 180$.

**Formule N°17:**
- En réalité, la développante est déphasée par rapport à la figure ci dessus.
- Pour exprimer ce déphasage, on calcule le paramètre angulaire $c$ au point où la développante coupe le cercle primitif.
- On a alors:
  - $zd(c)^2 + yd(c)^2 = rp^2$
  - $rb^2 \cdot (1 + c^2) = rp^2$
  - $\cos(a)^2 \cdot (1 + c^2) = 1$
  - $c^2 = 1/\cos(a)^2 - 1$

2. Start and configure the generative shape design workshop

- The part design workshop is not sufficient for designing parametric curves.
- So, we switch to the generative shape design workshop:

Next, we configure the environment for showing parameters and formulas:
- We set the 2 highlighted check boxes:
3 Enter the parameters and formulas

3.1 Define the primary generation parameters

- Switch to the Generative Shape Design workshop and click on the \text{F1}\text{ }] button:

- Then you can create the gear generation parameters:
  1. Select the unit (integer, real, length, angle, …).
  2. Press the create parameter button.
  3. Enter the parameter's name.
  4. Set the initial value, used only if the parameter has a fixed value.
3.2 Define dependent parameters

- Most of the geometric parameters are related to $a$, $m$, and $z$.
- You don't need to assign them a value, because Catia can compute them for you.
- So, instead of filling the initial value, you can press the **add formula** button.
- Then you can edit the formula.
3.3 Check the primary and computed parameters

- Set the following option in order to display the values and formulas of each parameter:
Now your tree should display the following parameters and their formulas:

3.4 Parametric laws of the involute curve

Up to now, we have defined formulas for computing parameters. Now we need to define the formulas defining the \( Y \) and \( Z \) cartesian position of the points on the involute curve of a tooth.

We could as well define a set of parameters \( Y_0, Z_0, Y_1, Z_1, \ldots \) for the coordinates of the involute's points. However, Catia provides a more convenient tool for doing that: the parametric laws.

In order to create a parametric law:

- click on the \( \text{fog} \) button:

Enter the formulas #14 and #15 of the 2 laws used for the \( Y \) and \( Z \) coordinates of the involute curve:

\[
\begin{align*}
y_d &= r_b \cdot (\sin(t \cdot \pi \cdot 1\text{rad}) - \cos(t \cdot \pi \cdot 1\text{rad}) \cdot t \cdot \pi) \\
z_d &= r_b \cdot (\cos(t \cdot \pi \cdot 1\text{rad}) + \sin(t \cdot \pi \cdot 1\text{rad}) \cdot t \cdot \pi)
\end{align*}
\]
Notes about the formula editor of Catia:
- The trigonometric functions expect angles, not numbers, so we must use angular constants like $\text{1rad}$ or $\text{1deg}$.
- $\pi$ stands for the $\pi$ number.

4. Create a geometric body and start inserting geometric elements

In Catia, the **PartBody** is intended for mechanical surfaces. For geometric constructions, you need to work in a geometric body:
- Create it with the **Insert / Open Body** top menu:
5. Make the geometric profile of the first tooth

The following steps explain how to design a single tooth. The whole gear is a circular repetition of that first tooth.

5.1 Define the parameters, constants and formulas
5.2 Insert a set of 5 constructive points and connect them with a spline

The position of each point is defined by the \( y(t) \) and \( z(t) \) parametric laws:

- Define 5 points on the YZ plane.

- In order to apply the involute formulas, edit the Y and Z coordinate of each point and enter the values of the parameter from \( t = 0 \) to \( t = 0.4 \) (most gears do not use the involute spiral beyond 0.4).
- For example, for the Y coordinate of the involute’s point corresponding to \( t = 0.2 \):

- Make a spline curve connecting the 5 constructive points:
5.4 Extrapolate the spline toward the center of the gear

Why do we need an extrapolation?

- The involute curve ends on the base circle of radius $rb = rp \times \cos(\alpha) = rp \times 0.94$.
- When $Z < 42$, the root circle is smaller than the base circle. For example, when $Z = 25$:
  
  \[ rf = rp - hf = rp - 1.25 \times m = rp \times (1 - 25 / Z) = rp \times 0.9 \]

- So the involute curve must be extrapolated for joining the root circle.

Extrapolate the spline:

- Start from the 1st involute point.
- The length to extrapolate is empirically defined by the formula $f(x) = 2 \times m$. 

5.5 Rotate the involute curve for the symmetry relative to the $ZX$ plane

Why do we need a rotation?

<table>
<thead>
<tr>
<th>Color</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RED</td>
<td>On the extrapolated involute curve designed in the $Y, Z$ coordinate system … the contact point on the pitch circle has an inconvenient position.</td>
</tr>
<tr>
<td>LIME</td>
<td>On the rotated involute curve … the two contact points of the tooth …</td>
</tr>
<tr>
<td>CYAN</td>
<td>that are located on the pitch circle at $\pm 90^\circ / Z$ …</td>
</tr>
<tr>
<td>MAGENTA</td>
<td>are symmetric relative to the $ZX$ plane.</td>
</tr>
</tbody>
</table>

- The colors above correspond to the following geometric elements:
For computing the rotation angle, we need first to compute the involute parameter or the pitch circle (formula #17):
Is it true? In order to check it, you can build two temporary elements:

- Insert another point and apply the involute formula with the parameter:

- Then, insert a half-circle having the radius of the pitch circle

- Check that the involute point with the parameter is actually located on the intersection of the pitch circle and the extrapolated spline curve:
Once the \( c \) parameter is checked, the temporary point and the temporary circle can be deleted.

Now, we can rotate the extrapolated curve, so that the first gear tooth is symmetric relative to the \( ZX \) plane:

- We use the formula \#18 for computing the \( \phi \) rotation angle in 2 steps:
  1. The curve is rotated by \( \tan( yd(c) / zd(c) ) \) so that the intersection between the involute and the pitch circle (the red point on the figure) is moved to the \( ZX \) plane.
  2. Then, curve is rotated by \( \frac{1}{4} \) of the gear period: \( 90 \text{deg} / Z \) (the left lime point on the figure), so that the \( ZX \) plane corresponds to the median plane of the first tooth.

A rotation operation is applied to the extrapolated spline, using the \( \phi \) rotation angle:
5.6 Draw the outer circle and the root circle

- We insert two half circles having a radius equal to \( r_a \) and \( r_f \), respectively.
- The figure below shows how to configure the outer circle:
5.7 Insert a rounded corner near the root circle

- The corner between the extrapolated involute curve and the root circle has a radius defined by the $r_c$ parameter.
- Catia asks you to select an arc (in red) out of 4 possible geometric solutions (in blue):
5.8 Create the rounded corner of the next tooth

Why are we going up to the next tooth?

- Initially, I designed a symmetric profile for the first tooth and I duplicated it \( z \) times:

\[
\text{But then, the generated profile was interrupted between each tooth by a fake edge:}
\]
For preventing that, I build now the whole profile between consecutive teeth on the root circle:

- On the figure above, you can see:
  - A vertical line tracing the $ZX$ plane.
  - An oblique line tracing the median plane between consecutive teeth.
  - This plane corresponds to the $ZX$ plane rotated by $\frac{180 \text{deg}}{Z}$ around the $X$ axis.

The following figure shows how to define that median plane:
Now, this plane is used for defining a symmetric rounded corner on the root circle:
5.9 Assemble the different elements of the first tooth

Now, we have to cut, fill and join the different elements of the 1st tooth:

- Cut the segment of the extrapolated spline between the outer circle and the rounded corner.
Define a symmetric profile relative to the $\text{ZX}$ plane, for the other side of the 1st tooth.
We could cut the root circle and the outer circle, but instead we define two arcs having a radius equal to $r_f$ and $r_a$, respectively:
The last operation consists in joining all the elements of the 1st tooth:
6. Build the whole gear profile and extrude it

The gear profile is just a circular repetition of the tooth:
- We define a repetition around the $X$ axis.
- The number of instances is controlled by the $Z$ parameter (number of teeth):
The first tooth and the duplicated teeth are joined for making the whole gear profile:
Now, we can switch back to the **part design workshop** (see the green arrow) and extrude the gear profile:
7. Cut the gear wheel

The gear wheel is cut after the extrusion, because each application requires a specific **wheel thickness**:  
- In a real factory, the teeth of the gear would be machined after the gear wheel is cut on a lathe.  
- In a CAD design, it is simpler to make the gear wheel with a groove, after the extrusion of the teeth.  
- That wheel design is semi-parametric: the external diameter and the 20deg chamfer are dependent of \( r_a \), but the bore diameter and the thickness are **adjusted manually** on the sketch:
Now, you can add pocket(s) for transmitting the torque between the gear wheel and a key or a splined shaft.

8. Check the parametric generation

Now you can play with the $Z$ and $m$ parameters and generate any spur gear:

- If $Z$ is equal to 13:
If \( Z \) is equal to 15: